Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

Ian Low, Fall 2008

Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #4

Due at 3:30 PM, October 24th

(One page and four problems.)

Reading Assignments:
Section 3.2 of Peskin and Schroeder.

Problem 1
Do Problem 3.4 in Peskin and Schroeder, but leave out part (e).

Problem 2
Do Problem 3.5 in Peskin and Schroeder.

Problem 3
(a) In the class we showed that the conserved currents corresponding to spacetime translations $x^\alpha \to x^\alpha - a^\alpha$ are the energy-momentum tensor $T^{\mu\nu}$. Since we have been considering Lorentz-invariant quantum field theories, derive the conserved currents corresponding to infinitesimal Lorentz transformations $\Lambda^\alpha{}_\beta = \delta^\alpha{}_\beta + \omega^\alpha{}_\beta$.
(Hint: recall that in the case of translations, there are really four currents $T^{\mu\alpha} \equiv (j^\mu)^\alpha$, one for each a^α. In this case there are really six conserved currents $M^{\mu\alpha}_\beta \equiv (j^\mu)^\alpha{}_\beta$, one for each $\omega^\alpha{}_\beta$. You may wish to express $M^{\mu\alpha}_\beta$ in terms of $T^{\mu\alpha}$.)
(b) What is the physical interpretation for each of the conserved charges in (a)? Separate your discussions into those corresponding to rotations and those corresponding to Lorentz boosts.

Problem 4
(a) A Lorentz transformation $\Lambda^\mu{}_\nu$ leaves the metric tensor $g_{\mu\nu}$ invariant: $\Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta \, g_{\mu\nu} = g_{\alpha\beta}$. Use this equation to prove that $\Lambda^0{}_0 \geq 1$ or $\Lambda^0{}_0 \leq -1$.
(b) Show that if two Lorentz transformations Λ_1 and Λ_2 both have $(\Lambda_1)^0{}_0 \geq 1$ and $(\Lambda_2)^0{}_0 \geq 1$, then $\Lambda_3 = \Lambda_1 \Lambda_2$ also has $(\Lambda_3)^0{}_0 \geq 1$. In other words, this sign is preserved under Lorentz group action and can be used to classify Lorentz transformations.
(c) Show that if two Lorentz transformations Λ_1 and Λ_2 both have $\mathrm{Det}(\Lambda_1) > 0$ and $\mathrm{Det}(\Lambda_2) > 0$, then $\Lambda_3 = \Lambda_1 \Lambda_2$ also has $\mathrm{Det}(\Lambda_3) > 0$. In other words, this sign is preserved under Lorentz group action and can be used to classify Lorentz transformations.
(d) Show all Lorentz transformations with $\mathrm{Det}(\Lambda) > 0$ and $\Lambda^0{}_0 \geq 1$ form a subgroup of the Lorentz group.